THE DOCTRINE OF INTERNAL RELATIONS:

RUSSELL'S 1897 REJECTION

Tyke Nunez

University of South Carolina

Abstract

Bertrand Russell's rejection of the doctrine of internal relations – the doctrine that all relations are determined by intrinsic properties of the terms related – was a critical step in his development away from his geometry-focused idealist philosophy of mathematics of the 1890s and towards his logic of relations and the logicism of *Principles of Mathematics*. Although this rejection is usually tied to his 1898 conversations with Moore (Hylton) or reading of Leibniz (Griffin), I argue that much earlier, in his 1897 *An Essay on the Foundations of Geometry,* Russell rejects the doctrine within geometry. I make this argument by examining his discussion of Hermann Lotze, Carl Stumpf, and William James in resolving what Russell calls "the antinomy of spatial relations." In this examination I argue that Russell rejects Stumpf's conception of spatial figures as grounded in an underlying "absolute content" (which is grounded in an acceptance of the doctrine of internal relations) in favor of a conception where spatial figures are merely relations. Russell finds support for this view in James. Even though Russell strives to stay neutral on the metaphysical issue of whether space is real or ideal, his *Foundations* view of space and the non-spatial atoms that underlie it is closer to Lotze's interpretation of Kant's view of phenomena and things in themselves than Lotze's own, which is grounded in the doctrine of internal relations. Tied to this, in closing I argue that reencountering Lotze's argument against Kant early in 1898 may have been one catalyst for Russell's warming to the doctrine of internal relations within geometry, before his rejection of it later that year. On the interpretation I'm offering, then, Russell rejects the doctrine in geometry in *Foundations*, comes to accept it in 1897 to 1898, before rejecting it again at the end of 1898, and in examining Russell's "early idealism," interpreters need to distinguish these fundamentally different positions.

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§1 – Introduction

In his early work on the philosophy of mathematics, before the development of his logic of relations and associated logicism, Russell took geometry to be the preeminent mathematical science and took the multiplicity essential to all mathematics to have space as its ultimate ground. The fullest development of this account comes in his 1897, *An Essay on the Foundations of Geometry* (hereafter: *Foundations*). In the fourth chapter Russell turns to developing his philosophical account of space through responding to issues that arise in the accounts of space and its representation given by Kant and Carl Stumpf.

A critical step towards what Whitehead called Russell's epoch-making development of the logic of relations (Whitehead 1902, p. 367), integral to his break with this geometrical philosophy of mathematics, was Russell's rejection of the doctrine of internal relations: the doctrine that all relations are grounded in and even determined by intrinsic properties of the terms related. [1] Russell's relationship to this doctrine was always a bit uneasy. As Hylton has stressed (1991, pp. 54–55), Bradley had argued that relations in general are afflicted with contradictions (e.g., Bradley 1893, ch. 3), and Russell clearly thinks relations require care (*Foundations*, §201, p. 193).[2] Still, before he rejects this doctrine Russell takes, in places, relational judgments to be analyzable into subject-predicate judgments, and seems to take such analyses to account for reasoning that depends on asymmetric transitive relations (e.g., 1898c, p. 225). Such relations underlie any ordering of elements and such orderings are critical to all of mathematics. Thus, Russell's motivation for developing a logic of relations depends on his first realizing that this kind of analysis is improper, and that the doctrine of internal relations is the problem.

How Russell realizes this is connected to what he comes to call in 1898 "the contradiction of relativity" (1898c, p. 166). We have this "contradiction," he thinks, whenever we have "a conception of difference, without a difference of conception" (1898c, pp. 258-9; see also 1898a, p. 132 and 1897b, p. 81). A "difference of conception" is a difference in the intrinsic properties of the related terms. If

the doctrine of internal relations were correct, then it seems that for every difference, there would be a corresponding difference in these intrinsic properties. Even while Russell accepts the doctrine, he sees that there is not always such a difference (witness: right and left hands). Hence the apparent contradiction.

It is a topic of some discussion when and why exactly Russell sours on the doctrine of internal relations, and so when exactly Russell thereby dissolves the contradiction of relativity. Peter Hylton (1990) and Nicholas Griffin (1991) have argued that it happens sometime in the second half of 1898, with Hylton stressing the influence of Moore, and Griffin focused on the evidence for the change in the Typescript of "An Analysis of Mathematical Reasoning" (1898) and the January 1899 "The Classification of Relations". More recently, although originally he had argued that Russell's reading of Leibniz has little to do with his rejection (1991, pp. 341–346), Griffin has argued that encountering the doctrine of internal relations in Leibniz was an important catalyst for Russell's rejection of it (2012). Finally, as this essay was headed to print, Griffin brought to my attention a forthcoming article downplaying the role of Leibniz and spelling out in more detail the exact influence of Moore (2022).

Jeremey Heis (2017) and Sanford Shieh (2019, ch. 5), however, have each argued that Russell is farther down the road to logicism in *Foundations* and in "An Analysis of Mathematical Reasoning" than interpreters like Hylton and Griffin acknowledge. Although this claim deserves important reservations, here I will argue that in the following respect they are right: we can already see skepticism about the doctrine of internal relations in *Foundations,* especially in its final chapter. Specifically*,* in *Foundations,* I will argue that we can see Russell is committed to rejecting the doctrine of internal relations within geometry. This commitment emerges in Russell's argument against Stumpf's view of space, and we will see that this rejection is bound up with a Kantian element in his *Foundations* view: his distinction between the form and matter of geometry. In this sense, we will see that the kind of commitment to – even heavy reliance on – the doctrine that Griffin's (e.g., 1991, pp. 146, 164, 187) discussion sometimes suggests is incongruous with the account in *Foundations*.

Still, two caveats are in order. First, in *Foundations,* the rejection of the doctrine of internal relations is clear only within the standpoint of geometry. From a wider standpoint that encompasses other sciences like psychology, at least in some contexts, Russell seems to accept the doctrine. Second, even within geometry, in work after *Foundations* but prior to "The Classification of Relations", like the

typescript of "An Analysis of Mathematical Reasoning*" or "On the Constituents of Space and their Mutual Relations", Russell seems to endorse a version of the doctrine. In this sense, I do not want to contest Hylton's and Griffin's claims about when Russell *decisively* abandons the doctrine of internal relations. Nonetheless, prior to "Constituents", in *Foundations,* there is an overlooked skepticism towards the doctrine. Recognizing this skepticism will make clear that between *Foundations* and "Constituents" Russell's view of space is radically and fundamentally transformed and that these two views should not be run together.

Specifically, although late in 1898 Russell seems to claim that what he had called "the antinomy of the point" in *Foundations* was an instance of the contradiction of relativity (1898d, p. 328-9n5), in *Foundations,* he does not, and would not, see it that way because, as we will see, he does not then accept the doctrine of internal relations in geometry. For this reason, it is no surprise that in *Foundations* we do not find him claiming that mere conceptions of difference without a difference in conception are contradictory. Rather, he seems to think they will be commonplace, and that they simply point to the importance of an immediate and intuitive source of knowledge (*Foundations*, §121, p. 131). On the reading I will here be developing, then, it is not that Russell merely generalizes the antinomy of the point into the contradiction of relativity in 1898. Rather, it is that between *Foundations* and 1898 he comes to accept the doctrine of internal relations, and through this reconceptualizes the antinomy of the point as an instance of the contradiction of relativity, in line with this fundamental transformation.[3] Then at the end of 1898 he rejects the doctrine along with idealism, which is what we find in COR. As a result, Russell's road to logicism is not a gradual progression, whereby he successively adopts more pieces of his logicist view, as Heis and Shieh can seem to suggest. Rather, his view undergoes at least one radical, fundamental transformation in his early idealist period when he switches from rejecting to accepting the doctrine of internal relations in geometry.

This essay is the first part of a larger project that will take a closer look at Russell's position in *Foundations*, which is both more interesting in its own right and more important for understanding the advent of modern logic than is commonly recognized. The view of space developed there is both quite Kantian and quite Leibnizian. Still, Leibniz endorses the doctrine of internal relations, while Kant rejects it, due to the problem of incongruent counterparts, at least within the phenomenal world of space and time. In the present essay Russell's relation to Leibniz's and Kant's views of space will remain largely off stage. Nonetheless, Russell's changing position on the doctrine of internal relations

is closely tied to his reflections on their conceptions of space, and the troubles that incongruent counterparts present that are connected to the contradiction of relativity, until his acceptance of the reality of asymmetric transitive relations in "The Classification of Relations". The larger project will delve into the Kantian and Leibnizian features of Russell's conception of space in *Foundations*, like his hylomorphism, as well as examine the early stages of the developments leading up to the logic of relations. Specifically, my hunch is that this examination will make clear that Russell's rejection of hylomorphism and his reflections on Kant's problem of incongruent counterparts played an unappreciated but critical role in the advent of Russell's logic of relations.

§2 – A sketch of Russell's *Foundations* view and our main text

We will focus on some of the final sections of the book where Russell is arguing that empty space should not be hypostatized. This is something that he thinks Newton, Kant, and Stumpf are all guilty of. Properly conceived, Russell thinks space is only the spatial order of things in it. To elaborate, briefly, Russell contrasts empty space with spatial order or figures. Such figures are what geometry studies. Fundamentally, according to Russell, they are relations between atoms, and he identifies spatial figures with spatially related atoms. These atoms, unlike spatial figures, are not themselves extended and are abstracted away from real physical atoms. Russell thinks of spatial relations, considered in abstraction from what they relate, as empty spaces. Now, although Russell takes spatial figures or order to be spatially related atoms, and so in this sense to be grounded in these atoms and spatial relations, in another sense he claims that spatial figures are more fundamental than the empty spaces that they occupy. In particular, he holds that we always begin with knowledge of spatial figures and then we derive knowledge of the space occupied from this prior knowledge of the figures. In this way, we arrive at a conception of such an empty space by abstracting away from the figures in it. When we do this in geometry, however, he maintains there is a strong temptation to hypostatize empty space and treat it as a thing, although "empty space itself, if it means more than the logical possibility of space-relations, is an unnecessary and self-contradictory assumption" (*Foundations*, §203, p. 194). So long as we do not hypostatize empty space, however, and regard space "so far as it is valid, as only spatial order", then the contradictions that afflict hypostatized empty space disappear (§205, p. 196).

In the second half of the fourth chapter, Russell is concerned with the resolution of three such contradictions that arise with the hypostatization of empty space. Our focus will be on the third, the

antinomy of spatial relations (§201, p. 193; §195, pp. 188–189). Prior to concluding his resolution of this antinomy in §207, he seems to endorse an argument of Lotze's against Kant's resolution of his mathematical antinomies (§205, p. 196). This apparent endorsement presents an immediate challenge to my thesis, because Lotze's argument can seem to rely on the doctrine of internal relations. Russell then considers and rejects Stumpf's view in §206 as subject to the antinomy of spatial relations. It is in this section, implicitly, that we find Russell's rejection of the doctrine of internal relations. For this reason, our focus will be on understanding it, and for this, it will help to have it in front of us:

> §206. But whence arises, on this [i.e., Russell's] view, the paradox that we cannot but regard space as having more or less thinghood, and as divisible *ad infinitum?* This must be explained, I think, as a psychological illusion, unavoidably arising from the fact that spatial relations are immediately presented. They thus have a peculiar psychical quality, as immediate experiences, by which quality they can be distinguished from time-relations or any other order in which things may be arranged. To Stumpf, whose problem is psychological, such a psychical quality would constitute an absolute underlying content, and would fully justify his thesis; to us, however, whose problem is epistemological, it would not do so, but would leave the *meaning* of the spatial element in sense-perception free from any implication of an absolute or empty space. [Footnote: "Cf. James, *Psychology*, Vol. II., p. 148 ff."] May we not, then, abandon empty space, and say: Spatial order consists of *felt* relations, and *quâ* felt has, for Psychology, an existence not wholly resolvable into relations, and unavoidably *seeming* to be more than mere relations. But when we examine the information, as to space, which we derive from sense-perception, we find ourselves plunged in contradictions, as soon as we allow this information to consist of more than relations. This leaves spatial order alone in the field, and reduces empty space to a mere name for the logical possibility of spatial relations. (§206, pp. 196–197)

Although we are not yet, of course, in a position to analyze this difficult and somewhat cryptic passage, notice that both Stumpf's view is Russell's main target here and the question that Russell begins with.[4] The context for this question is the prior section, §205. For this reason, it makes sense for us to begin there in the next section (§3). Because this is also where Russell endorses Lotze's argument, it will provide an occasion to discuss the most direct evidence in *Foundations* that Russell might endorse the doctrine of internal relations. After this, we will turn to considering the features of Stumpf's view that Russell is referencing in §4. That will put us in a position to return to this passage and examine how Russell is arguing against Stumpf and rejecting the doctrine of internal relations within geometry in §5. Finally, we will examine the two caveats I mentioned above in §6, where we will see that Russell's hylomorphic *Foundations* view has an affinity to how Kant's view of phenomena and things in themselves was understood by Lotze, and that reencountering Lotze's critique of Kant's hylomorphism may have been part of the catalyst for Russell's rejection of the *Foundations* view early in 1898, although he already identifies the contradiction of relativity in 1897.

§3 – Russell's endorsement of Lotze's argument

It is in Russell's brief dismissal of Kant's resolution to his mathematical antinomies in §205 that we find Russell coming closest to endorsing the doctrine of internal relations in *Foundations*:

> Another ground for condemning empty space is to be found in the mathematical antinomies. For it is no solution, as Lotze points out (*Metaphysic*, Bk. II. Chap. I., § 106), to regard empty space as purely subjective: contradictions in a necessary subjective intuition form as great a difficulty as in anything else. But these antinomies arise only in connection with empty space, not with spatial order as an aggregate of relations. For only when space is regarded as possessed of some thinghood, can a whole or a true element be demanded. This we have seen already in connection with the Point. When space is regarded, so far as it is valid, as only spatial order, unbounded extension and infinite divisibility both disappear. (§205, p. 196)

Kant's mathematical antinomies, remember, are these:

> 1st Thesis: The world has a beginning in time, and in space it is also enclosed in boundaries.
>
> 1st Antithesis: The world has no beginning and no bounds in space, but is infinite with regard to both time and space. (A426–427/B454–455)
>
> 2nd Thesis: Every composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed of simples.
>
> 2nd Antithesis: No composite thing in the world consists of simple parts, and nowhere in it does there exist anything simple. (A434–435/B462–463)

In the latter part of the just-quoted *Foundations* passage, Russell argues that his own account is not subject to these antinomies because on his view, "unbounded extension and infinite divisibility both disappear" since space is only the order of things in space. Without addressing whether he is right, on its face this at least seems in line with the sketch of Russell's view above: he rejects unbounded empty space, and spatial order is ultimately an order among indivisible simple atoms.

Turning to Lotze's criticism of Kant that Russell endorses in the former part of the passage, in these sections of his *Metaphysic* (1884), the first thing to notice about Lotze's discussion is that it begins with a defense of various features of Kant's account of space. Specifically, in §103–104 Lotze endorses many aspects of the account of space that emerge in Kant's Transcendental Aesthetic, and he claims that the arguments here do not disagree in a substantive way with the impressions of everyday life (§104, pp. 180–181). It is in the antinomies, "which seemed to result from [space's] presupposed relation to the real world", however, that Lotze thinks "the motives to such a startling transformation

of the ordinary view were found by Kant" (§105, p. 181). This transformation is Kant's claim that space-perception is "*nothing but* a subjective form of apprehension". Perhaps surprisingly, Lotze goes on to argue that Kant is still right about this (esp. §114–115), even though he complains that Kant's antinomies do not establish space to be purely phenomenal because this "does not properly speaking remove any of the difficulties" (§105, p. 181). He explains:

> there must be determinations in the realm of things in themselves prescribing the definite places, forms, or motions, which we observe the appearances in space to occupy, sustain, or execute [...] If Things are not themselves of spatial form and do not stand in space-relations to one another, then they must be in some network of changeable intelligible relations with one another; to each of these, translated by us into the language of spatial images, there must correspond one definite space relation to the exclusion of every other. (§105, pp. 181–182)

For this reason, Lotze holds that "it is quite inadmissible" to "treat Things in themselves as utterly foreign to the forms under which they were nevertheless to appear", as is "the fashion especially of popular treatises of the Kantian school" (p.181). He maintains that Kant "has left unanswered" how "particular apparent things find their definite places in" the "innate and consequently uniform perception of space" (p.182). Still, Lotze argues that developed in the way that he defends, Kant is correct that space is merely phenomenal, and is right to distinguish spatial objects from the real things in themselves that lie at their root (§116, p.199).

In §106 Lotze then discusses Kant's first antinomy, and in §107, Kant's second. In each he defends the thesis and rejects the antithesis. He maintains the thesis in each case is not actually "mysterious or suspicious" (p.184), but that even so, if the phenomenal world, say, has a finite extent, or is composed of simple physical atoms, then because "there must be precisely as many distinguishable elements in the world of things in themselves as there are different points of space in the world of perception", the noumenal world too will have a finite extent and be composed of simples (§106, p. 182). Consequently, "space could only possess its empirical reality if there were conceded to the real world [of things in themselves] that very countlessness or infinity the impossibility of admitting which was the reason for restricting space to an empirical reality" (§106, p. 182).[5]

On Lotze's view in these passages, then, there is a non-spatial world of things in themselves that grounds the phenomenal world. The phenomenal world is constituted through the relation of things in themselves to our forms of sensibility, space and time, and so the objects of this world, appearances,

are themselves relations. Now, although things in themselves are not spatial, these things have intrinsic properties that are sufficient to ground all of the spatial properties of appearances in the phenomenal world. Thus, so long as things in themselves are considered in relation to our forms of sensibility, we have appearances, which stand in spatial relations to one another. Lotze (like Russell, Kant, Stumpf, and Leibniz) views space as a system of relations, and he holds that positions in space are fixed, and have unchangeable spatial properties that determine their spatial relations. Things in space, appearances, occupy these positions, and their spatial relations are determined through the positions they occupy. Nonetheless, each appearance, A, corresponds to a thing in itself, X. And for every spatial property s of A, he takes there to be some non-spatial property n of X that grounds s. That is, although spatial properties of appearances are determined through their positions, there will always be a property of the thing in itself grounding the appearance that determines the position that this appearance is in. Thus, Lotze thinks there is always some property, n, of the thing in itself, X, that grounds every spatial property, s, of the appearance, A, and these in turn are what account for the spatial relations of appearances (compare §116, p. 199). In this sense, he seems to maintain that all spatial relations of phenomenal things are ultimately grounded in (1st) intrinsic spatial properties of an appearance, A, and that these spatial properties are grounded in turn in (2nd) intrinsic non-spatial properties of a thing in itself, X.

This sounds like it is built on an endorsement of the doctrine of internal relations both vis-à-vis the relational spatial properties of appearances and the corresponding intrinsic properties of things-in-themselves. This is because it both treats spatial relations as grounded in intrinsic spatial properties of appearances, and views appearances as themselves relations whose spatial properties are grounded in non-spatial properties of things in themselves. There is evidence that Russell would have thought of it this way in "The Classification of Relations". Here Russell decisively rejects the doctrine of internal relations and here it is Lotze who mainly comes in for criticism (1899, pp. 144–145). But does Lotze endorse the doctrine?

Lotze sums up the results of his discussion of relations as follows: all "relations" "only exist either as ideas in a consciousness which imposes them, or as inner states, within the real elements of existence, which according to our ordinary phrase stand in the 'relations'" (1884, §109, p. 187).[6] In this way, all "relations" end up either being projections by some conscious subject, or being reducible to intrinsic properties of the relata, so that properly speaking no relations in the ordinary sense remain. Thus, the

things and their relations in space are in a conscious subject, but they have at their root a "network of changeable intelligible relations" between things in themselves. Nonetheless, according to Lotze, these intelligible relations will really end up being a collection of inner states or properties of the intelligible things.[7] All of which strongly suggests that Lotze endorsed a version of the doctrine of internal relations.[8]

Now, from *Foundations* §205 alone, it is not clear how we should hear Russell's approving reference to Lotze's argument. It could be taken to involve a full-scale endorsement of the doctrine of internal relations and of the deployment of it against Kant through its applicability to appearances and things in themselves. Perhaps, however, Russell's citation of Lotze's critique of Kant's response to the antinomies is not meant that way. Perhaps Russell intended only to agree that viewing space as a subjective intuition isn't enough to get Kant out of trouble, so long as the contradictions that afflict hypostatized empty space persist in that intuition. In this case, the approving reference indicates Russell's repeated claim that appeals to psychology will not help resolve the issues with which he is concerned, but it would not be an endorsement of the doctrine of internal relations. To see why Russell's own view forecloses the strong interpretation and why we should prefer this weaker reading we will have to take account of the argument that Russell makes against Stumpf.[9]

§3 – Stumpf's absolute content and the feeling of a spatial quality

§206 opens with the questions: "But whence arises, on this view [i.e., Russell's], the paradox that we cannot but regard space as having more or less thinghood, and as divisible *ad infinitum*" (*Foundations*, §206, p. 196)? We can now see that these questions are tied to how Russell reads Kant's mathematical antinomies. After all, whether or not there is a whole of space that we regard as having thinghood or whether or not in dividing space we continue *ad infinitum* are, respectively, each akin to the first and second antinomies. Remember, according to Russell, if we keep in mind that space is really only spatial order, and that empty space only indicates the possibility of spatial order, then the temptation to see space as an empty whole or as infinitely divisible disappears. This is because we only have an existing infinite whole or an existing infinite division if we have mistaken possible space for actual spatial order and hypostatized empty space.[10] But why are we tempted to do this? In answering these questions, Russell maintains that the reason we think we must regard space as both having thinghood and as divisible *ad infinitum* is due to a psychological illusion "unavoidably arising from the fact that spatial

relations are immediately presented. They thus have a peculiar psychical quality, as immediate experiences, by which quality they can be distinguished from time-relations or any other order" (§206, p. 196).

Russell elaborates his account of this psychological illusion through the contrast between the views of spatial representation of William James and Carl Stumpf. Stumpf was a prominent psychologist and philosopher, remembered most today for advising Husserl's habilitation and for being a student of Brentano and Lotze.[11] Understanding the argument of §206 depends on understanding Stumpf's notion of an "absolute content", and to introduce this, it will help to look briefly at the section Russell cites from James, titled "Space Relations".

In this section James argues that spatial order, which encompasses perceptions of "figures, directions, positions, magnitudes, and distances" are "qualities of sensation" (1890, pp. 147–148). Unlike most relations, James claims that "*in the field of space the relations are facts of the same order with the facts they relate*" (p. 149; compare *Foundations*, §172, p. 171). He thinks we can see this because the feeling that accompanies space relations, unlike the feeling that accompanies other relations, is the same in kind as the feelings that accompany what the relation relates (1890, p. 149). James seems to be thinking that there is a common quality to the perception of any spatial relation similar, say, to the quality common to every perception of red, or common to every perception of a bolt of cloth, or common to every perception of a book. In seeing a red cloth and a red book, however, we might think of the sensation of seeing a cloth and the sensation of seeing a book as terms that are related insofar as they are both accompanied by the sensation of seeing red. In this case the feeling of seeing red is heterogeneous with the feelings of seeing a book or cloth. In the case of space-relations, however, James seems to be thinking that the sensation that accompanies seeing place A, and the sensation that accompanies seeing place B, is the same in kind as the sensation of seeing the relation between A and B, because this relation of A and B is itself a place.

Although we will see that in other respects James's view contrasts with Stumpf's, Russell, not implausibly, attributes a similar view to Stumpf. Before turning to this, we should note that the fundamental commitment of Stumpf's *On the Psychological Origin of the Representation of Space* (1873) (*Über Den Psychologischen Ursprung Der Raumvorstellung*) is that we can represent space only by representing qualities in space. When we attempt to represent space without any qualities, we can not help but

imagine it filled with, say, blackness or whiteness, etc. Stumpf takes this to be in contrast with Kant, who claims that "Space is a necessary representation, *a priori*, that is the ground of all outer intuitions. One can never represent that there is no space, though one can very well think that there are no objects to be encountered in it" (A24/B38–39). Here Stumpf reads Kant as maintaining that "we can think away the qualities, but not space" and that this means we can think of space without color, but not color without space (1873, p. 19). He maintains that the psychological claim this presupposes is wrong. We can not represent space at all without qualities such as color or hardness, nor can we represent such qualities at all without representing space.[12]

When Russell attributes something like James's view to Stumpf, he claims that "[t]o Stumpf, whose problem is psychological, such a [homogeneous spatial] psychical quality would constitute an absolute underlying content, and would fully justify his thesis" (*Foundations*, §206, p. 196). Wrapped in this is an attribution of commitment to the doctrine of internal relations. To see why, we will need to see what is meant by "an absolute underlying content". The closest Russell comes to giving an explanation is the following quote from Stumpf:

> There is no order or relation without a positive absolute content, underlying it, and making it possible to order anything in this manner. Why and how should we otherwise distinguish one order from another? [...] To distinguish different orders from one another, we must everywhere recognize a particular absolute content, in relation to which the order takes place. And so space, too, is not a mere order, but just that by which the spatial order, side-by-sideness distinguishes itself from the rest. (1873, p. 15)[13]

Although Russell elides them, to illustrate his case, Stumpf gives the following examples:

> We can order a library according to the size of the books, the color of their covers, their publication dates, their content, etc.; we can order six people according to their moral sensibilities, their intellectual capacities, their bodily strength; among others, we can also order them according to the time in which they live, and the place in which they find themselves. (1873, p. 15)[14]

Stumpf's thought is that a positive absolute content is supposed to underlie any ordering and is supposed to allow for the distinction of one order from another. He seems to hold that orderings are something we create and distinguish, but that there are properties of the things ordered that allow us to do this. The examples he gives are what allow for ordering in terms of size, color, date, content, moral sensibility, intellectual capacities, strength, time, and place. In each of these cases there is some

feature had by each book or person that allows it to be ordered in relation to the other books or people. An absolute content seems to be a collection of such features of things that allow their ordering. So, for example, the sizes of books might be an absolute content of the books in the library, because every book has its size, and all of the books can be ordered according to their size. Similarly, say, human beings will each have a moral sensibility, and Stumpf's thought seems to be that we can order people according to this property. In this case, an absolute content would be a collection of features of things that are suitably homogeneous such that they can ground an ordering of the things. On the interpretation I am proposing, then, the absolute content is not the ordering relation, nor this relation and its relata, but the potential relata with the features that allow their ordering.

In the case of spatial content, I think we can see that this reading of absolute content is borne out. Specifically, Stumpf seems to consistently treat places (*Orte*) as absolute contents, and to distinguish these from their ordering. For example, he disambiguates between "location [*Lage*]" as a "relation of places" (*örtliche Beziehung*) and location as "place" (*Ort*), in which case it is not a relation, "as follows from the fact that [a point] *b* can change its place without [a point] *a* changing its own" (p. 124).[15] Because Stumpf holds that every order or relation must have an absolute content grounding it, we should expect that spatial place relations like location and distance should be grounded in places, which are the absolute contents that underlie them. This is exactly what we find him arguing when he turns to the nature of "extension or magnitude" (*ausdehnung oder Grösse*). He takes these to be relations between the parts of a place that we think along with any place – specifically, they are the difference between the place's outermost parts (p. 280).

> That place is the origin of both concepts shows itself in that something can change its place without changing its magnitude, while every change in its magnitude partially changes its place along with it. An absolute content can alter, while its internal relations stay the same, but no relation can change without the absolute content that lies at its basis experiencing some change. (p. 280, also p. 150)

Just as we can order the books in a library according to their size, when they were published, their topics, etc. because there is something about the books that grounds all of these various orderings, there is something in places that allows us to order them according to their different relational properties. So just as Kathryn is shorter than Matt because she is sixty-five inches tall and he is seventy-two inches tall, places *a* and *b* have properties like height that are their own individually, but that determine, say, that *a* is below *b*.

What are these features of places that are like the individual heights of Kathryn and Matt? Late in his *Raumbuch*, Stumpf clarifies how he is thinking about the intrinsic spatial properties that ground the order of things in space. He points out that although a relation without absolute content is impossible, an absolute content cannot be conceived of completely independently of all relations (p. 181). He holds that spaces must have a center point, that all points in the space are situated in relation to it, and that the absolute content of a place is determined through its relation to such a center point (e.g., pp. 180–182, pp. 283, 307).[16] Specifically, he holds that "the relation of places, *a*, *b*, *c*, to a center, *C*, is something different from their relations to one another. Place *a* can be represented on its own, alone, without relation to *b* or *c*, but not without relation to *C*" (p. 181). The reason for this asymmetry seems to be that because a given space will have a center point, we can assign a coordinate system to the whole space and indicate any place in the space through its coordinates. These coordinates, then, can serve as features that, together with the coordinates of other places, determine the spatial relations that they stand in. Places *a* and *b* being situated in relation to the center point *C* of a line, so that *a* is at -1 and *b* is at 2 would be taken to be sufficient to ground *a*'s standing to the left of *b*, and similarly for other spatial relations. In this way, then, the coordinates of places depend on their relation to a center point. And these coordinates are the intrinsic properties of the places that ground all of their further spatial relations.

Because Stumpf takes every relation to be grounded in an absolute content, and such a content is a collection of intrinsic properties had by the things that will be related, it seems he would have endorsed the doctrine of internal relations for space.[17] After all, just as the taller-than relation is grounded in the intrinsic heights of different people, according to Stumpf, all spatial relations are grounded in the intrinsic spatial coordinates of the positions of the things in question.

Nonetheless, his view also includes a significant exception to the doctrine. Places are defined through their *relation* to a center point. Because of this, it is through this original relation that the intrinsic coordinates of the places are fixed, according to Stumpf. So although spatial relations are for the most part grounded in the intrinsic spatial features of positions, these intrinsic features themselves – the coordinates of positions – are grounded in this prior, original relation to a center point. For our purposes, it is significant that Russell does not mention this exception. For this reason, I suspect he

would likely take Stumpf to fully accept the doctrine, even though he was well aware of the special role of an origin point in assigning coordinates within a system (compare, e.g., *Foundations*, §42, p. 43).

Returning to Russell's claim that to Stumpf "such a [homogeneous spatial] psychical quality would constitute an absolute underlying content" (§206, p. 196), we now can get a better sense of what he had in mind. Remember, on Stumpf's view it is a psychological fact that we cannot represent space without representing a quality that fills it, like blackness or hardness. Setting aside qualities specific to a sense, however, it seems that there will also be a common psychical quality to the felt spatial relations that positions have to a center point. This sensation of distance from a center point is the fundamental kind of sensation that grounds all of the other sensations of spatial relations that are endemic to the representation of space. So as James suggested, for Stumpf too, when we represent spatial properties like "__ is to the left of - -", this will always be accompanied by a certain sensation that is grounded in the intrinsic properties of positions, which are fixed through the sensation of their distances from a center point. These sensations will be homogeneous. And it is because Stumpf takes each position to have intrinsic properties of distance from a center point, which are independent of such properties of other positions, and because these intrinsic properties ground their relations, that Russell takes Stumpf to endorse the representation of space as an absolute underlying psychical content. Thus, as Russell reads James, his point about the homogeneity of spatial sensation can be marshaled in favor of Stumpf's conception of space as an absolute underlying content, except in grounding this homogeneity the psychical quality of 'standing at some distance from a center point' takes on the role for Stumpf that the quality of 'having some extent' had for James.

§4 – Russell's reply to Stumpf and the rejection of the doctrine of internal relations

Let us turn, now, to the problem that Russell finds in Stumpf's view in the second half of §206. While "to Stumpf, whose problem is psychological", according to Russell, "such a psychical quality would constitute an absolute underlying content, and would fully justify his thesis", for his own epistemic purposes, Russell claims that such a spatial sensation or psychical quality

> would leave the *meaning* of the spatial element in sense-perception free from any implication of an absolute or empty space. [Footnote: "Cf. James, *Psychology*, Vol. II., p. 148 ff."] May we not, then, abandon empty space, and say: Spatial order consists of *felt* relations, and *quâ* felt has, for Psychology, an existence not wholly resolvable into relations, and unavoidably *seeming* to be more than mere relations. But when we examine the information, as to space, which we derive from sense-perception, we find

ourselves plunged in contradictions, as soon as we allow this information to consist of more than relations. (*Foundations*, §206, pp. 196–197)

We will see that Russell is doing two things in this passage. First, he is pointing to a conflict between Stumpf and James within psychology over the need for an absolute content. This conflict is a version of the antinomy of spatial relations under discussion in §201–§208, and it is this that we will introduce next. Second, he is claiming that although this conflict is an illusion endemic to psychology, in epistemology we can avoid it, so long as we do not allow that the information derived from sense perception consists in an absolute content. Psychology is afflicted with the antinomy of relations because it, in part, treats spatial figures as grounded in the intrinsic features of spatial positions in accord with the doctrine of internal relations, but the epistemology of space escapes this antinomy by rejecting the attempt to ground spatial figures in intrinsic features of their terms. This is why the doctrine of internal relations turns out to be inapplicable in geometry.

Taking up the antinomy of space relations, Russell comes at it through a point he finds in Bradley: "on the one hand, space has parts, and is therefore not mere relations, while on the other hand, when we try to say what those parts are, we find them after all to be mere relations" (§202, p. 194). Here his thought seems to be that relations do not have parts. Space has parts. So space is not mere relations. Yet space seems to be mere relations, because when we examine the parts of space we only find more spaces, and spaces are mere relations. That this is what Russell has in mind is even clearer in Russell's introduction of the antinomy: "spatial figures must be regarded as relations. But a relation is necessarily indivisible, while spatial figures are necessarily divisible *ad infinitum*" (§195, p. 189). Here he seems to be thinking that a spatial figure is regarded as a relation. A spatial figure, e.g., a line segment "—" is divisible into parts: "-" and "-" and the whole line segment can be thought of as a relation between its two parts. Nonetheless, a relation is indivisible. Why? Well, a relation relates relata. For example, "loves" relates in "Othello loves Desdemona". The relation "love", however, does not divide into parts. As a result, it seems spatial figures cannot be relations.

After introducing Bradley's version of the antinomy, Russell wonders, could the space which has parts "be regarded as empty space, Stumpf's absolute underlying content, which is not mere relations, while the parts, in so far as they turn out to be mere relations, are those relations which constitute spatial order, not empty space" (§202, p. 194)? In §202 he goes on to suggest that this is a promising (but

obscure) possibility; however, when he returns to Stumpf in §206, he seems to have soured on this solution. This is puzzling. Why in §206 does Russell take the antinomy of spatial relations to afflict the account of space given by Stumpf? And how come Stumpf can not avoid the antinomy in the way suggested in §202?

The answers seem connected to the reference to James. We saw above that James's point about the homogeneity of the sensation or feeling that accompanies space relations can be marshaled in support of Stumpf's claim that space is an absolute underlying content. Nonetheless, Russell cites James in the service of rejecting Stumpf's absolute contents. What explains this? The answer is evident in the rest of James's section on space relations:

> *The line is the relation*; feel it and you feel the relation, see it and you see the relation; nor can you in any conceivable way think the latter except by imagining the former (however vaguely), or describe or indicate the one except by pointing to the other. And the moment you have imagined the line, the relation stands before you in all its completeness, with nothing further to be done. […] *The relation of position between the top and bottom points of a vertical line is that line*, and nothing else. (1890, pp. 149–150)[18]

Here, the line, or any spatial figure, is the relation of the points that compose it. The feeling of the line or figure is identical to the feeling of the relation. Imagining or conceiving of the line involves imagining or conceiving of the relation. And conceiving of the line is nothing else but conceiving of the relation.

What, however, is involved in conceiving of the relation? If Stumpf's view is right, then conceiving of a line or spatial relation involves not only conceiving of the relation between two points, but also conceiving of the absolute space that underlies it – the relata that ground the relation. Thus, while James thinks conceiving of the line is nothing more than conceiving of the relations of the points, Stumpf thinks that this also involves something more: conceiving of the absolute space underlying it. Both views, Russell thinks, are justified by psychology. "Spatial order consists of *felt* relations" and this is all spatial figures seem to be (*Foundations*, §206, p. 197). Yet because they are felt, they also have "an existence not wholly resolvable into relations, and unavoidably *seeming* to be more than mere relations" (§206, p. 197). Thus, we have a psychological version of the antinomy of spatial relations because spatial figures both *feel* like they are, and *feel* like they are not, wholly resolvable into relations.

The solution Russell suggests, at least for epistemology, is to keep the homogeneity of space, while cutting out appeals to sensations and feeling. He takes them to be the source of the psychological illusion. Instead, he focuses on the *information* that we derive from sense-perception. Once we do this, he thinks we are free to treat spatial order as nothing more than the mere relations of things in space. If we treat this information as though it consisted in something more, like Stumpf's absolute contents, then he thinks we will be plunged into contradictions. Specifically, he thinks we will be subject to the antinomy of spatial relations, as well as Kant's mathematical antinomies. If we ignore the psychological illusion, reject Stumpf's absolute space, and treat empty space as "a mere name for the logical possibility of spatial relations", then he claims that we can avoid the antinomies (§206, p. 197).

This is the strategy he then goes on to develop in his resolution of the antinomy of spatial relations in §207. In it he argues that the "divisibility of the relations which constitute spatial order" is merely apparent (§207, p. 197). With a forgivable anachronistic use of notation, take a line, $L(\_\_, - -)$, which is a relation with two argument places, indicated by '$\_\_$' and '$- -$', that relates two atoms, A and B: $L(A, B)$. $L(\_\_, - -)$ will be a relation that relates the relata A and B. Russell's point in §207 is that we often confuse $L(A, B)$ with $L(\_\_, - -)$, and that this is the source of the antinomy. This is because, while $L(A, B)$ is divisible, $L(\_\_, - -)$ is not. For example, we might imagine that the line $L(A, B)$ was divided by another atom, C. Now instead of the original line $L(A, B)$, we have $L'(A, C)$ and $L''(C, B)$, which consist in the relations $L'(\_\_, - -)$ and $L''(\_\_, - -)$ relating the atoms, $A, C,$ and B. Still, "the original relation is not really divided: all that has happened is, that two or more equivalent relations have replaced it, as two compounded relations of father and son may replace the equivalent relation of grandfather and grandson" (§207, p. 197). In this way, although the original relation $L(\_\_, - -)$ may seem to have been divided into $L'(\_\_, - -)$ and $L''(\_\_, - -)$, because $L(A, B)$, has been divided into $L'(A, C)$ and $L''(C, B)$, $L(\_\_, - -)$ has not actually been divided at all. Rather, it has been replaced by two new relations: $L'(\_\_, - -)$ and $L''(\_\_, - -)$, just as two instances of "$\_\_$ is the father of $- -$" might replace "$\_\_$ is the grandfather of $- -$" without the second relation being divided. In this way, although the original line segment is divisible into two further line segments, because spatial order is homogenous, the empty relation that constitutes this line segment is not itself divisible.

Wrapped into Russell's resolution of the antinomy of relations is a rejection of the doctrine of internal relations. This is especially clear once one considers that even though the atoms stand in "spatial relations to other elements" and are the ultimate terms of the relations that constitute spatial figures

(§199, p. 192), they are "non-spatial simple" elements (§199, p. 192), "have no intrinsic spatial adjectives" (§200, p. 193), and are themselves unextended. After all, if Russell accepted the doctrine of internal relations within geometry, then spatial figures would be grounded in intrinsic properties of the atoms that they relate. Geometry, however, only studies spatial properties. So because these are unextended "non-spatial simple" elements, as far as geometry is concerned, they are identical. Thus, there is no way that spatial figures are reducible to the properties of their atoms, and Russell must reject the doctrine of internal relations within geometry.

So far, however, Russell's rejection for epistemology of space as an absolute content and of the doctrine of internal relations may look *ad hoc*. Rejecting these gives Russell his resolution of the antinomy of spatial relations, but can Russell give a further reason why epistemology is not afflicted with the same illusion as psychology? That is, why maintain that the psychological sensation or feeling of spatial figures presents them as both merely relations, and as more than mere relations, while also maintaining that the *information* we derive from sense perception about space does not present them this way, but only as spatially ordered terms?

A preliminary answer might be that if we set aside the immediacy of sense perception and only think about the nature of spatial figures as spatial relations, we will see that an underlying absolute spatial content is superfluous. Spatial figures, as related relata, are *Thises* (returning to Russell's Aristotelean turn of phrase, §187–9, pp. 182–184; §199, p. 192; §204, p. 195). What makes a *This* a *This*, on an Aristotelean view, is its form.[19] Russell here takes that form to be the relation (compare §188, p. 183). The material, the atoms, themselves are potential relata. They do not make the figure what it is, but merely provide the occasion for the relation, the form, to make a *This* (compare §199, p. 192).[20] Thus, epistemically, there is no need to treat the relata as themselves containing intrinsic properties that ground the relation, as though the matter of the *This* was itself sufficient to determine its form and what the *This* is. In this respect, given Russell's Aristotelean appeals to geometrical *Thises*, which consist in a form and a matter, it is no surprise that he rejects the doctrine of internal relations for geometry. If we were to delve further into Russell's account of space and his notion of a "form of externality", I believe that we would find more evidence for this position because such a form both has the role of allowing the differentiation of positions as external to one another and of grounding the relations between these positions, like $L(\_\_, - -)$.[21]

There is another clearer reason that epistemology should not view space as an absolute content although psychology should, on Russell's view. This has to do with the reason why Stumpf claims that every space has a center point. Remember, Stumpf claims that all spatial representation must have a quality or color. Closely connected to this, he takes the fundamental, original spatial representation to usually be of a visual field. Such a field always has a subject whom it belongs to, and who is situated at its center. When we construct spaces, he thinks we naturally place our subject at the center of the space, and we determine the location of every outer thing in the space in relation to it (1873, pp. 306–307). Nonetheless, we can abstract from the "here" of our subject and the origin of this center point in us. After all, in geometry, our subject and the orientation of our bodies in space is not a topic. Still, Stumpf holds that we cannot separate a space from its center point altogether (p. 180). When we imagine any space – even just the one dimensional space of a line – he claims that we will include a center point and we will locate points positively and negatively in relation to it (pp. 307, 180). This essential inclusion of a coordinate system that determines direction and orientation is, then, a vestige of the psychology of the spatial situation of our subject at the center of our visual field.

Russell cannot accept this conclusion. If Stumpf were right, then representing space would require representing it as having a determinate metric. Projective geometry, however, depends on representing space without representing the metric of that space. Thus, if Russell is to give an account of the possibility of projective geometry, as he aspires to do, he must reject Stumpf's conclusion. Furthermore, Stumpf's argument is clearly psychological: it concerns first and foremost the psychology of spatial representation. If we consider only the *information* that we get about *space* from spatial perception, then nothing follows about the center point of that space. After all, if you and I are in the same room, the way that we center our respective visual fields will differ, yet no difference between the spaces that we are in follows from this difference in our visual fields. Thus, it is not unreasonable for Russell to conclude that in epistemology, we need not presume that every space has a center point, and that every position in space has intrinsic coordinates determined by their distance from this center point. For this reason, if epistemology can avoid positing that things in space have such intrinsic spatial properties, that would serve to delineate it more clearly from psychology. And in this respect, Russell's rejection of Stumpf's view of space as an absolute content is not a merely *ad hoc* solution to the antinomy of spatial relations, but is critical to his attempt to separate psychology from epistemology and geometry, and to his overall project in *Foundations*.

§5 – Caveats: the return of the doctrine of internal relations and its place outside geometry

I introduced the doctrine of internal relations as the doctrine that all relations are grounded in intrinsic properties of the terms related. We have now seen that in geometry, in *Foundations*, Russell rejects this doctrine. In closing, I would like to return to the two caveats I mentioned at the outset. First, following Stumpf, Russell takes the atoms that ground geometry to be real physical atoms, except we have abstracted from all of their causal properties. This suggests that although within geometry Russell thinks we should treat them as intrinsically propertyless, in reality they have properties that differentiate them. From this wider standpoint, Russell might not have rejected the doctrine of internal relations. Second, looking past *Foundations* to Russell's work shortly after, we find him endorsing a modified version of the doctrine even in geometry. In this period, we will see that he seems to endorse a view quite close to Stumpf's or Lotze's. I will close by speculating on why Russell changed his view.

As to the first caveat, in *Foundations,* Russell introduces possible atoms to replace points. With the introduction of atoms, he is moving beyond the narrow standpoint of geometry, and considering the nature of space from a broader philosophical one. This kind of shift in standpoint is characteristic of Russell's neo-Hegelian dialectic of the sciences. According to this dialectic, the special sciences have an order. Within the standpoint of a given special science, one will encounter contradictions endemic to its subject matter. Resolving these contradictions necessitates considering further determinations of this subject matter. That is, it requires moving from considering the things in question more abstractly (say, spatially) to considering them more concretely (say, physically).

At the close of *Foundations*, we get an example of such a progression. There Russell indicates that the unextended atoms, or matter, that he has introduced in order to resolve the spatial antinomies themselves will face new antinomies, but to deal with these "would demand a fresh treatise, leading us, through Kinematics, into the domains of Dynamics and Physics" (§209, p. 201). In this way, we begin with geometry, an abstract science of space and possible space relations. We see that the basic posits of this science, points and empty spaces, lead to contradictions. We resolve these contradictions by introducing extensionless atoms into geometry, the possibility of which can replace points and the relation of which are spatial figures. This "matter in general" belongs to geometry (§209, p. 200), but is nonetheless abstracted from the matter of physics (§197, p. 190), and introducing it may constitute a move to 'kinematic geometry', if we can make sense of such a science. [22] While geometrical matter is

"not regarded as possessing any causal qualities, as exerting or as subject to the action of forces", the matter of physics, on the other hand, is subject to such action (§199, p. 191). So while the partial view afforded by geometry is contradictory, by moving to physics we can resolve those contradictions, but then we must contend with new features of the matter in question, forces and causal qualities, which will in turn lead to new contradictions. [23]

Now, even while writing *Foundations,* there is evidence that Russell did not think that the doctrine of internal relations should always be rejected in all sciences. In a note from early 1897, when commenting on the "transition from punctual matter to the plenum" and the antinomy of absolute motion which is a contradiction he finds in kinematics, he remarks that "the true view seems to be Lotze's" (1896–1898, p. 22). And earlier in the same notebook when considering "dynamics and absolute motion" in a note from 1896, Russell indicates that the way to resolve these contradictions in physics may be to define position by reference to axes or a coordinate system. That gives "hope in restoring the preëminence of the *here*, as a source of absolute position", and perhaps we may replace "force by conation, and pass on into psychology" as the last stage in the dialectic, before the general all-encompassing science of metaphysics (p. 16).

Although Russell does not develop the plan from these notes, and it is hard to see how psychology might be less contradictory than other sciences, these remarks are not at odds with *Foundations.* After all, in §206 we saw that Russell attempts to forestall the effects of a psychological illusion within geometry and epistemology. Specifically, he rejects as too psychological Stumpf's attempt to ground an absolute coordinate system on the *here* of the subject. That is consistent, however, with the preëminence of the *here* being a source of absolute position in psychology. Still, we also saw that within psychology, Russell thought Stumpf's absolute contents, as well as James's rejection of them, could both be justified, and it is not clear how Russell would have attempted to reconcile the contradictory views of Stumpf and James.[24] Nonetheless, if we set the contradictory nature of psychology to one side, and consider just the legitimacy of the doctrine of internal relations, then in §206 Russell does seem to think this doctrine is applicable, at least in some contexts. After all, he thought Stumpf was justified in appealing to the doctrine. So although it remains obscure how a limited appeal to the doctrine is supposed to help with overcoming the contradictions of psychology, at the time of *Foundations,* Russell seems more open to the doctrine outside of geometry than within it.

Turning to the second caveat, early in 1898 Russell takes the doctrine of internal relations to be applicable to spatial figures and comes around to a view that is much closer to Stumpf's or Lotze's. In *Foundations* Russell had already noticed that within geometry we often have, as he later puts it, "a conception of difference, without a difference of conception" (1898c, p. 226). Although he is untroubled by these cases in *Foundations* (e.g., *Foundations*, §121, p. 131), later he comes to think they are instances of the "contradiction of relativity". It is in elaborating this contradiction in the typescript of an "Analysis of Mathematical Reasoning" from mid 1898 that we find the following endorsement of the doctrine of internal relations.

> We have things with definite spatial or temporal relations, and these relations do not confer the same adjective on both the terms. The difference of adjective is expressed in Mathematics by difference of sign; this expresses, for example, the difference between *A's* adjective of being east of *B,* and *B's* adjective of being west of *A*. We must suppose, therefore, that any intrinsic properties of *A* and B corresponding to the spatial relation must be different. We assume such intrinsic properties to rationalize the relation, and call them positions. Thus *A* has one position and *B* another: since the relation between *A* and *B* is not reciprocal, the two positions should differ. But when we examine either position *per se,* it is found to have no properties but what are shared by all positions. To make the positions of *A* and *B* different, as they must be, we have to take account of the relation: then *A* becomes a position which is east of *B,* and *B* becomes a position which is west of *A*. Thus we have here again a conception of the difference—*A* differs from *B* by lying west of it—but we have no difference of conception. […] We are supposed here to have two terms *A* and *B,* with a relation *R* which transforms them into *Aβ* and *Bα*. *β* is an adjective which has a reference to *B,* and *α* similarly has a reference to *A*. Neither can be expressed without this reference, and *α* and *β* differ in content. But *A* and *B,* considered without reference to the relation *R,* have no differences of conception corresponding to the differences *α, β*. (1898c, p. 225; compare Lotze, 1884, p. 199)

Here the contradiction of relativity is that *A* and *B* are in different positions, and so the intrinsic properties of their positions should be different, yet these intrinsic properties are the same.

Notice that in this passage Russell claims the positions of the terms *A* and *B* are identical in themselves *per se*. Nonetheless, *A* and *B* are *transformed* by the relation *R*. Once transformed, *A* acquires the property of *β*, or of being to the east of *B*; and B acquires the property of *α*, or of being to the west of *A*. This can seem like a rejection of the doctrine of internal relations, and not so different from *Foundations*. There we saw that qualityless atoms are related geometrically through the space that they inhabit. This is because the atoms of *Foundations,* as the non-spatial elements of geometry, are intrinsically identical. Thus, it can not be that their intrinsic properties ground their spatial relations. Rather, these are grounded first and foremost in the relation, or the space to which this relation belongs. *In this relation*, these atoms acquire geometrical properties like "being to the east of A" or

"being to the west of B", or α and β. In this respect, although in *Foundations* unrelated atoms, or unenformed matter, are intrinsically identical, *after* two atoms are related through space, they acquire properties like α, β, which are grounded in their relation. In this sense, in *Foundations* too, related atoms, or enformed matter, can have properties that are grounded in the relation.[25] And for that reason, the break between the two views can seem small.

The substantive break with *Foundations,* however, lies at the beginning of the "Analysis" passage, with its proposal that there are *intrinsic* properties of A and B corresponding to the spatial relation. That is incompatible with the atoms of *Foundations*. To see why, suppose that the related atoms, or the enformed matter, of *Foundations* had intrinsic properties corresponding to their spatial relations. The non-spatial, unrelated, unenformed atoms*,* however, are supposed to be identical with these spatially related atoms. Thus, intrinsically considered, in isolation from whatever relations they stand in, the unrelated and the related atoms should be identical. If the related atoms have intrinsic properties that correspond to their spatial relations, however, then they cannot be identical. After all, the non-spatial atoms are non-spatial because they are supposed to be devoid of such intrinsic spatial properties. Thus, there is a substantive break between the *Foundations* and "Analysis" views when it comes to the properties of the terms ultimately related.

This break is evident in a March of 1898 preparatory study for "Analysis" titled "On the Constituents of Space and their Mutual Relations". In *Foundations,* the external relations of a figure "constitute its position" (§124, p. 133). External relations can distinguish a figure from others, but a change in its external properties can "in no way affect its internal properties" (§124, p. 133). In this sense, position is an extrinsic property of figures, not an intrinsic one. "Constituents of Space", however, opens with the claim that "every point has a quality peculiar to itself, called its position. This quality is intrinsic, and is that by which points differ" (1898b, pp. 311, 320). "Constituents" goes on to attack the *Foundations* position that the basic terms of spatial figures are themselves non-spatial (pp. 317–318), and contra *Foundations,* it defends the claim that geometry cannot but hypostatize its relations and rejects the view that everything spatial is merely relational. Finally, although this essay does not contain an endorsement of the doctrine of internal relations that is quite as developed as the one from "Analysis", in a footnote he distinguishes relations in a "narrow sense" where "it is impossible to find, in any of the related terms, any adjective corresponding to the relation and yet not stated with a

reference to the other terms of the relation" (p. 316n.). "All spatial relations", including distance, belong to this class, about which Russell claims:

> [T]he relation may be formally exhibited as an adjective of either term, but it is an adjective in which the reference to the other term cannot be omitted. We assume, to rationalize this relation, an intrinsic property, position, by which the two points differ; but as pointed out above, nothing can be said about position *per se*, but only about differences of position. (p. 316n.)

This endorsement of intrinsic adjectives of the terms in a relation that "rationalize the relation" is quite close to the kind of endorsement of the doctrine of internal relations that we found in "Analysis". In both cases intrinsic adjectives make essential reference to another term. Thus, in both cases we have the oxymoron of an *intrinsic* adjective with an essentially *extrinsic* reference. Perhaps this problem is not deep – perhaps there is no trouble in the *existence* of an intrinsic property that can be indicated or *known* only through an external reference – but it should still make us wonder: why does Russell come to prefer a view that endorses such paradoxical properties to the view of *Foundations*?

In the "Constituents of Space", Russell gives the following argument against taking non-spatial atoms to be the basic constituents of spatial figures. He claims that a view where a non-spatial something underlies spatial figures treats points as "mere content" (i.e., as properties or relations), not things (p. 317). Russell holds that this view "makes everything spatial mere content" (p. 317). He claims that on it each point does not *have* an (intrinsic) quality – position – but *is* the quality of position by which *things* are distinguished. This seems to fit with the *Foundations* view of geometry and epistemology. On it, *things* have positions by which they are distinguished, but points themselves do not have intrinsic qualities of position. The problem with such a view, according to "Constituents", is "that each point is a unique quality, which can, by its essence, only belong to one existent. Thus it is not a true content, but, as we might say, an *existent* adjective. It is not, in Bradley's sense, a wandering adjective, and is not, as we require it to be, a *mere* content" (p. 318). Russell's thought here seems to be that if spatial figures are mere relations of non-spatial things, points or positions are supposed to be qualities that allow the distinction of these things. On this view, points, along with everything else spatial, are supposed to be mere contents. Contents, however, can belong to more than one thing, while points cannot. So positions are not mere contents.

This is not an easy argument and to plumb its depths would require diving deeper into the radically altered view of "Constituents" and the changes leading to it than we can here.[26] We can make some progress, however, by addressing an objection that might be bothering the reader. I have argued that in *Foundations,* geometry is grounded ultimately in atoms, and that these are non-spatial. In part on these grounds I have argued that Russell would reject the doctrine of internal relations in geometry. Lotze, however, endorsed this doctrine. He held, remember, that for every spatial property, s, of an appearance, A, there must be some non-spatial property, n, of a thing in itself, X, that grounds s. Things in themselves were essentially non-spatial. Thus, in both cases the ultimate ground for spatial figures or appearances lies in something that is essentially non-spatial, and the inference from the non-spatiality of Russell's atoms to the rejection of the doctrine of internal relations looks invalid.

Notice, however, the large difference between Russell's atoms and Lotze's things in themselves. Russell's atoms abstract away from all of their features except for their bare difference from one another. For the purposes of geometry, the atoms are matter in general, lacking all further intrinsic properties. Their spatiality arises through their relations and prior to this, they are indistinguishable. They are merely an occasion for their differentiation and relation through space, which is the source of the distinctive features of spatial figures. This is similar to how we saw that Lotze reads Kant, or at least treatises of the Kantian school, since such atoms leave unanswered how "particular apparent things find their definite places in" the perception of space (1884, pp. 181–182). In contrast, for Lotze, things in themselves contain the ground of all their possible properties. It is through their relation to us that appearances in space arise. Although there is nothing spatial about those things themselves, they have properties that correspond to the spatial properties of the appearances they ground that distinguish them from one another. In this way, although on both Russell's and Lotze's views, atoms and things in themselves ultimately ground geometrical figures, Russell's atoms are baren and empty, at least as they are considered by geometry, while Lotze's things in themselves are fecund and infinite, containing the intrinsic ground of all their possible relational properties. So while it is correct that non-spatiality on its own does not rule out reliance on the doctrine of internal relations, this is ruled out by stripping out intrinsic properties from the atoms that ground spatial figures and taking the distinctive features of these to be grounded in space.

Returning to the "Conditions" argument, there the problem with the *Foundations* view was at least in part supposed to be that it treated points or positions as general properties that could be occupied by,

and thus common to, many things. In fact, on the "Conditions" view, however, points are existent adjectives or qualities that are absolutely particular, and occupying different points is sufficient to distinguish two things. In this way, in "Conditions" Russell is shifting to a view of points where they have absolutely distinctive properties. As a result, rather than being indistinguishable and propertyless, on the "Conditions" view, points and positions are more like Lotze's property-rich things in themselves, except that their properties are spatial. In this respect, in "Conditions", Russell comes to think of one legitimate view of space as similar to the absolute space of Stumpf. On this view, space is the sum or aggregate of all of the qualitatively, intrinsically differentiated points and positions.

Thus, points and positions can be legitimately viewed as hypostatized existents, according to "Conditions". In *Foundations,* Russell thought that hypostatizing space in this way was the source of contradictions in geometry, but that these contradictions could be avoided by positing atoms and beginning the transition to physics. In "Conditions", this hypostatizing of space is no less contradictory (1898b, pp. 318, 320–321). But now Russell thinks this hypostatization is unavoidable, and that a dilemma quite close to the antinomy of spatial relations is "ultimate and insoluble" (p. 321). If in "Conditions" the contradictions accompanying hypostatization seem even more inevitable, we might ask, again, why does he change his view?

In closing, let me point to one further remark that might help explain the shift. In his notes on McTaggart's 1898 Lent Term lectures on Lotze, which would have been around the time he wrote "Conditions" (although well after his identification of the contradiction of relativity in "On the relations of number and quantity"), we get what might be another clue. In a brief discussion of a contrast between Lotze and Kant that corresponds to the one between *Foundations* atoms and Lotze's things in themselves, Russell (2020, p. 64) writes: "Lotze's doctrine improvement on Kant: avoids Kant's duality. Form must have some connection with matter". This echoes Lotze's (1884, p.181) claim that Kant, or treatises of his school, "treat Things in themselves as utterly foreign to the forms under which they were nevertheless to appear". We saw that because the atoms of *Foundations* are baren, Lotze would take them to be incapable of explaining the spatial properties of the appearances they ground. And here we see that he seems to think this lack of explanatory power entails that the matter is utterly foreign to, or unconnected with, the form of space.

Of course, it is hard to say whether Russell endorsed the claim of these notes or whether he is only faithfully recording the lecture. There is no parenthetical commentary of the sort that he often makes in his own voice. Nonetheless, the remark is suggestive. We saw above that on Aristotelean hylomorphic views, unenformed matter does not constitute a *This* because it is a thing's form that makes it what it is. Unenformed matter is merely potentially a *This*. And on such views there is no expectation that unenformed matter determines what a *This* would be. This is not a bug; it is a feature. In these notes, however, Russell seems to worry with Lotze that unless the unenformed matter and its features explains the enformed matter and its features, this leads to a duality that cleaves matter from form so that no connection remains. If, however, points and positions were existent qualities that were intrinsically differentiated from all other points and positions like on Stumpf's view, then this material of geometry would already be intrinsically spatial, points would already be intrinsically tied to the space they occupy, and thus, space as form would have a connection with matter. In this way, by early 1898 Russell had clearly broken with the hylomorphism of *Foundations*.[27]

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[1] As Russell (1959, p. 42) later puts it: "The doctrine of internal relations held that every relation between two terms expresses, primarily, intrinsic properties of the two terms and, in ultimate analysis, a property of the whole which the two compose". This is a Russellian construal of the doctrine. Another that is primarily found in Moore holds that on the doctrine no relations are contingently true, which is treated as entailing their necessity. Also, a commitment closely associated with the doctrine is that relations are ideal, or that they are less real than their relata. Neither this, nor Moore's construal of the doctrine will be our focus. We will be restricting our attention to the Russellian version.

[2] For a recent reassessment of the dispute between Russell and Bradley see Candlish (2007). Chapter 6 treats relations.

[3] A useful exchange with Jim Levine helped me formulate this way of presenting my view of Russell's development.

[4] Also worth an initial mention is the contrast between epistemology and psychology that Russell is drawing in the passage. Russell is primarily concerned with these in the context of representing space, and the psychology of spatial representation was a hot topic in the second half of the 19th century. (For background, see Hatfield (1990).) We will see that in *Foundations*, and specifically in the passages we will be examining, Russell is developing his own conception of this distinction.

[5] It is a bit opaque how Lotze reconciles this apparent pluralism about things in themselves with his monism, according to which "Things can only exist as parts of a single Being, separate relatively to our apprehension, but not actually independent" (1884, §69–71, p. 125–128). Ultimately, it seems that even the distinctions between things in themselves must be only distinctions that we draw, but which are not really in the things themselves (compare, e.g., §98, p. 172–173).

[6] Here he cites his discussion of relations in section §81, where he imagines relations as analogous to threads between their relata, say, *a* and *b*. These threads communicate "a definite tension" to both, so that *a* and *b* are thereby in a different inner condition than they would be if they were not so related (1884, §81, p. 144). But because of this difference in their inner condition, he holds that "the termini *a* and *b* can produce *immediately* in each other these reciprocal modifications" (§81, p. 144). And because of these immediate modifications, he holds "that there is no such thing as this interval between things" and so we can get rid of the thread or relation, which thereby "subsists not between things but immediately in them as the mutual action which they exercise on each other and the mutual effects which they sustain from each other" (§81, p. 144). (Thanks to Daniel Sutherland for pushing me to present more evidence that Lotze endorses the doctrine of internal relations.)

[7] If we bear in mind that ultimately the distinctions between intelligible things will be distinctions among the parts of one monistic reality, however, then in a way this is not very surprising, because "relations" between intelligible things will indicate intrinsic properties of this one reality.

[8] Lotze may even be a main source of Russell's later formulation of it. In his discussion of relations Lotze (1884, §80–81) touches on all three aspects of the doctrine that I mentioned in footnote 1. He endorses not merely the view that every relation between two terms expresses, primarily, intrinsic properties of the two terms, but also the closely associated doctrine that relations are ideal and are less real than their relata. It is less clear, however, that he holds all relations are not contingently but necessarily true, since in §81 it is such contingent relations that he is most concerned to argue reduce to intrinsic properties of their relata (compare, §81, p. 143). Overall, however, in what ways Leibniz, Lotze, Stumpf, Brentano, Bradley, Bosanquet, or Ward, etc. may be the source of the doctrine I will leave to the speculations of others.

[9] Although there is not space to develop this thought here, the cost of the weaker reading is that Russell does not otherwise spell out why he thinks Kant is guilty of hypostatizing empty space. Without the stronger reading it is unclear why Russell held Kant couldn't avoid the antinomies in a way that mirrored Russell's own solution by treating things in themselves in the way that Russell treats his non-spatial atoms. Russell charges Kant with making illicit appeals to psychology. Showing that this thought is correct would involve showing that these allegedly psychological elements wouldn't block Kant from treating things in themselves like Russell's non-spatial atoms.

[10] Russell's use of "divisible *ad infinitum*" is a bit tricky. As I read him, Russell takes the actual space of spatially ordered things to both be indefinitely extendible and indefinitely divisible. In both cases these processes will not reach an end, and

so long as we are talking about these *processes*, it is not a problem to say that space (qua spatial order) is divisible *ad infinitum*. The problem is thinking that space is already infinitely divided or infinitely extended. That is what Russell would not allow.

[11] Although more attention has been paid to Stumpf as a member of the "Brentano school" (e.g., Pradel, 2015; Chrudzimiski, 2015), there is work on the influence of Lotze on Stumpf (e.g., Centi 2011; Milkov, 2015; Textor 2020). The ideas of both Brentano and Lotze were in circulation in Cambridge in the 1890s. Specifically, Bell (1999), Preti (2008), and van der Schaar (2013, 2017, 2018) have examined the influence of Brentano on Moore through Stout and Ward. Milkov (2020, ch. 6 & 7) has argued that the "Hegelianism" in the air in Cambridge was more Lotze than Hegel, and has pointed to the influence of Lotze not only on Russell, but also on Green, Stout, Ward, Bradley, and Bosanquet. Levine (2018) gives a nice, succinct presentation of the "idealist" influences on Russell, with a focus on McTaggart, Bradley, and Ward. Stumpf's role, through his conception of space, as a conduit of Brentano's and Lotze's ideas to Russell deserves further investigation.

[12] This is a thought that Stumpf finds already in Berkeley and Hume (1873, pp. 23–24). Textor (2018) and MacCumhail (2020) offer recent discussions of Stumpf's account of perception. Textor focuses on its unity, while MacCumhail focuses on this fundamental commitment.

[13] I have given Russell's translation (*Foundations*, §202, pp. 193–194). The illustration that immediately follows is what Russell elides. The translation there is mine, as are all other translations of Stumpf.

[14] Stumpf seems to take the library simile from Lotze, whose "local signs" are like the labels on books in a packed-away library (compare Stumpf 1873, p. 86; Stumpf 1891, p. 486n; Lotze 1856, pp. 334–335). Stumpf seems to be appropriating Lotze's simile in order to convince his reader that Lotze's theory of local signs and spatial order can be effectively presented through the notion of an absolute spatial content (1873, pp. 85–86).

[15] The attentive reader might find Stumpf's claim that points have places jarring, as the discussion of James above is plausibly read to imply that places are extents, and that the qualitative homogeneity between places and the relations of places depends on this. I sympathize. Although Stumpf considers unextended points to be "unrepresentable abstractions" only made for "scientific purposes" (1873, pp. 280, 58), he does seem to think of them as places, since he sometimes specifies that he means extended, not punctiform, places (p. 280). Shortly we will see that for Stumpf the homogeneous spatial psychical quality of places has a different origin than it did for James, such that not only extents but points share it.

[16] Although points seem to be places for Stumpf, a space is a collection of places, and the representation of a space involves the representation of all of the spaces in between two places (1873, p. 16), so points alone do not seem to be spaces.

[17] Chrudzimski (2015, §4) claims that the whole Brentano school has a strong tendency towards this doctrine, and traces it back to Aristotle. There are suggestive passages in both Aristotle and Plato (e.g., *Meta*, 1088a20–b4; *Phaedo*, 102b–e), but these are not decisive endorsements of the doctrine. For a recent discussion of relations in Aristotle and Plato, see Duncombe (2020); for a classic discussion, see Simplicius (2002). (Thanks to Josh Mehndolson for conversation on this, and for pointing me to the Duncombe and Simplicius). For a number of historically informed contemporary discussions of the metaphysics of relations, see Marmodoro and Yates (2016).

[18] In his discussion of distance Russell quotes this passage and offers some discussion of it (*Foundations*, §172, p. 171; compare also the marginalia in his copy of James's *Psychology*, 1997, p. 153). Thanks to Alex Klein for conversation about James.

[19] See, for example, *Meta*, *VII* (Z) *& VIII* (E). In some places Aristotle clarifies that a "this" is a "this such", where the "such" indicates its form and makes the thing *what it is*, but the matter makes the thing this individual thing and not some other individual thing of the same kind (e.g., *Meta*, 1033b25).

[20] Notice, although Russell claims in this passage that the atom "must be a *This* not resolvable into *Thises*", because it is not extended, if it is a *This*, it is not a *This* in the same sense as a spatial figure is a *This*. Indeed, the unrelated atom seems to be like unenformed matter. And as with such matter, one might wonder whether it is a *This* at all.

[21] Russell's notion of a "form of externality" is closely related to Kant's notion of a "form of intuition". Like how space according to Kant is the form of outer intuition, or immediate perception, Russell considers spaces of various curvatures or dimensionalities to be possible forms of externality (e.g., *Foundations*, §57, p. 57). For both Kant and Russell these forms are homogeneous, and the homogeneity of space and its parts is not grounded empirically in a feeling, but is an *a priori* feature of it and the form of intuition or externality through which it is known. (For a discussion of the importance of homogeneity for Kant's conception of space and geometry, see Sutherland (2021, esp. ch. 7).) This modification of Kant's notion is closely related to Russell's adaptation of Kant's distinction between form and matter. He aims to adopt the epistemic or logical side of Kant's distinction, but reject its supposedly psychological side (compare §59, p. 62, §183, pp. 180–181). Developing a detailed interpretation of Russell's hylomorphism in this period, however, would take us well beyond the doctrine of internal relations. It is, thus, a project that must wait for another occasion.

[22] Thanks to Nick Griffin for conversation on this point.